## 1. Beadható BEDTACI.ELTE

$$L\left(\bigcup\_{i=1}^{n}L\_{i}\right)=\bigcup\_{i=1}^{n}LL\_{i}$$

Elindulok mindkét irányból, míg össze nem érnek az egyenlőségek.

$$L\left(\bigcup\_{i=1}^{n}L\_{i}\right)=L\left(L\_{1}∪L\_{2}∪…∪L\_{n}\right)=LL\_{1}∪LL\_{2}∪…∪LL\_{n}$$

$$\bigcup\_{i=1}^{n}LL\_{i}=LL\_{1}∪LL\_{2}∪…∪LL\_{n}$$

## Véges példa

$$L=\left\{aa,bb\right\}$$

$$L\_{1}=\left\{aa,b\right\}$$

$$L\_{2}=\left\{ab,ba\right\}$$

$$L\_{3}=\left\{ab,ϵ,aaba\right\}$$

$$L\_{4}=∅$$

$$L\_{5}=\left\{ϵ\right\}$$

$$L\left(\bigcup\_{i=1}^{n}L\_{i}\right)=\left\{aa,bb\right\}\left\{aa,b,ab,ba,ϵ,aaba\right\}=$$

$$=\left\{aaaa,aab,aaab,aaba,aa,aaaaba,bbaa,bbb,bbab, bbba,bb,bbaaba\right\}=$$

$$=\left\{aa,aaaa,aaaaba,aaab,aab,aaba,bb,bbaa,bbaaba,bbab,bbb,bbba\right\}$$

$$\bigcup\_{i=1}^{n}LL\_{i}=\left\{aa,bb\right\}\left\{aa,b\right\}∪\left\{aa,bb\right\}\left\{ab,ba\right\}∪\left\{aa,bb\right\}\left\{ab,ϵ,aaba\right\}∪\left\{aa,bb\right\}∅∪\left\{aa,bb\right\}\left\{ϵ\right\}=$$

$$=\left\{aaaa,aab,bbaa,bbb\right\}∪\left\{aaab,aaba,bbab,bbba\right\}∪\left\{aaab,aaaaba,bbab,bbaaba\right\}∪\left\{aa,bb\right\}∪\left\{aa,bb\right\}=$$

$$=\left\{aaaa,aab,bbaa,bbb,aaab,aaba,bbab,bbba,aaaaba,bbaaba,aa,bb\right\}=$$

$$=\left\{aa,aaaa,aaaaba,aaab,aab,aaba,bb,bbaa,bbaaba,bbab,bbb,bbba\right\}$$

## Végtelen példa

$$L=\left\{a^{n}b|n\in N\right\}=\left\{b,ab,aab,aaab…\right\}$$

$$L\_{1}=\left\{ab^{n}|n\in N\right\}=\{a,ab,abb,abbb…\}$$

$$L\_{2}=\left\{c^{n}|n\in N\right\}=\left\{ϵ,c,cc,ccc…\right\}$$

$$L\_{i}=\left\{d^{i}\right\}; i\in N$$

$$L\left(\bigcup\_{i=1}^{n}L\_{i}\right)=\left\{b,ab,aab…\right\}\left\{a,ab,abb…,ϵ,c,cc…,ϵ,d,dd…\right\}=$$

$$=\left\{ba,bab,babb,…,aba,abab,ababb,…,aaba,aabab,aababb,…,b,bc,bcc,…,ab,abc,abcc,…,aab,aabc,aabcc,…,b,bd,bdd,…,ab,abd,abdd,…,aab,aabd,aabdd,…\right\}$$

$$\bigcup\_{i=1}^{n}LL\_{i}=\left\{b,ab,aab…\right\}\left\{a,ab,abb…\right\}∪\left\{b,ab,aab…\right\}\left\{ϵ,c,cc…\right\}∪\left\{b,ab,aab…\right\}\left\{ϵ\right\}∪\left\{b,ab,aab…\right\}\left\{d\right\}∪\left\{b,ab,aab…\right\}\left\{dd\right\}∪…=$$

$$=\left\{ba,bab,babb,…,aba,abab,ababb,…,aaba,aabab,aababb,…\}∪\{b,bc,bcc,…,ab,abc,abcc,…,aab,aabc,aabcc,…\}∪\{b,ab,aab,…\}∪\{bd,abd,aabd,…\}∪\{bdd,abdd,aabdd,…\right\}∪…=$$

$$=\left\{ba,bab,babb,…,aba,abab,ababb,…,aaba,aabab,aababb,…,b,bc,bcc,…,ab,abc,abcc,…,aab,aabc,aabcc,…,b,ab,aab,…,bd,abd,aabd,…,bdd,abdd,aabdd,…\right\}$$

(A sorrend ugyan más, de egyébként megegyeznek.)

## Általánosítás

$$L\left(\bigcup\_{i=1}^{n}L\_{i}\right)=L\left(L\_{1}∪L\_{2}∪…∪L\_{n}\right)=LL\_{1}∪LL\_{2}∪…∪LL\_{n}=\bigcup\_{i=1}^{n}LL\_{i}$$